

Home Work III: Lecture 11-15; due by Tuesday April 22, 2008

- Two km thick dust storms are moving all year long from West Africa to the Caribbean islands 5000 km away. The concentration being measured in the Caribbean is $C=10 \times 10^{-6} \text{ g.m}^{-3}$. The dust storms are transported by the easterlies at an average speed of 7 m.s^{-1} . We can assume that dust mass distribution is log-normal with a mass median diameter of $5 \times 10^{-6} \text{ m}$, and a geometric standard deviation=2. The dynamic viscosity of air is $1.7 \times 10^{-5} \text{ kg.m}^{-1}.\text{s}^{-1}$ and dust density is 2500 kg.m^{-3} .
 - By expressing the deposition velocity as $v_d=kd^2$ as a function the square of the particle diameter d , gives the values of k in units of $\text{m}^{-1}.\text{s}^{-1}$.
 - Write the equation of a Lagrangian box model for dust mass distribution with gravitational settling (expressed by kd^2) as the only loss term.
 - Calculate the time needed for the dust storm to reach the Caribbean.
 - Calculate the initial value $C(t=0)$ by integration of the box model equation over the particles diameter d .

ANSWER

1.a. The settling viscosity is proportional to the product of d^2 and $k = \frac{1}{18} \frac{\rho_p g C_c}{\mu}$ where $\rho_p=2500$

kg.m^{-3} is the aerosol density, $g=9.81 \text{ m.s}^{-2}$, $C_c \sim 1$, and $\mu=1.7 \times 10^{-5} \text{ kg.m}^{-1}.\text{s}^{-1}$ is the dynamic viscosity. The value of $k=8.2 \times 10^{-7} \text{ m}^{-1}.\text{s}^{-1}$

1.b. The only flux component through a Lagrangian box of dust over the ocean is the vertical gravitational settling. The continuity equation is given by

$H \frac{\partial C}{\partial t} = -v_d C$, which has the analytical solution $C(d,t) = C_0 \exp(-\frac{tkd^2}{H})$ where C_0 is the initial concentration.

1.c. The traveling time is given by the ratio $5000\text{km}/7 \text{ m.s}^{-1}$, which corresponds to 8.3 days.

1.d. Assuming a log-normal distribution, the concentration C can be expressed as

$$C(d,t) = \int_0^\infty \frac{C_0}{\sqrt{2\pi r \log \sigma}} \exp\left(-\frac{1}{2} \frac{\log^2(r/r_v)}{\log^2 \sigma} - \frac{t}{H} k(2r)^2\right) dr$$

Where $r_v=2.5 \times 10^{-6} \text{ [m]}$ is the volume mean radius and $\sigma=2$ is the standard deviation.

For a mono-disperse distribution of particles of radius r_v , the initial concentration is

$$C_0 = C \exp\left(\frac{tk4r_v^2}{H}\right) = 10^{-5} \exp\left(\frac{5 \times 10^6 \times 8.2 \times 10^{-7} \times 4 \times (2.5 \times 10^{-6})^2}{2 \times 10^6}\right) = 2.1 \times 10^{-5} \text{ g.m}^{-3}$$

By assuming that the median volume radius is constant in time, the log-normal distribution can be integrated over the radius, and the value of $C_0 = 3.17 \times 10^{-5} \text{ g.m}^{-3}$.

2. Answer to 5 of the 8 questions

1. Explain briefly with words the advantages and drawbacks of measuring backscatter radiances in the visible and near-Ultraviolet to detect aerosols with satellite instrument.

ANSWER:

Advantages: 1) low reflectivity of all land surface types (including the normally bright deserts in the visible), which make possible aerosol retrieval over the continents, 2) sensitivity to aerosol types that absorb in the UV, allowing the separation of carbonaceous and mineral dust aerosols from purely scattering particles.

Drawbacks: 1) Vertical profile of aerosols needs to be provided to retrieve quantitative results. This is due to Rayleigh scattering which is not negligible in the UV and is depending on air density or pressure

2. How does MODIS algorithm retrieve aerosol optical thickness in the visible although without knowing the surface reflectivity in the visible?

ANSWER

Aerosols are assumed transparent in the nIR, which allows retrieving the surface reflectivity in nIR. Using linear relationships between surface reflectivity in nIR and at different wavelength in the visible, it is then possible to retrieve the aerosol optical depth.

3. What type of additional information the TOMS algorithm needs to retrieve aerosol optical thickness and single scattering albedo?

ANSWER

TOMS algorithm uses two the measurement of backscatter radiances at 2 wavelength in the nUV to retrieve 2 optical properties. Due to strong molecular scattering in the nUV, there is an altitude dependency of the measurements, and the aerosol vertical profile needs to be obtained from additional datasets.

4. Collection 5 MODIS data are supposed to provide better retrieved aerosol optical thickness compare to collection 4 MODIS data. What are these improvements when comparing with AERONET data?

ANSWER

Comparison of MODIS collection 4 and 5 with AERONET data shows data the latest algorithm which has improved the retrieval of surface reflectivity (e.g. by considering NDVI) has corrected a high and low bias at low and high aerosol optical depth, respectively.

5. What does the diffusion term represent physically in the mathematical formulation of aerosol transport?

ANSWER

The diffusion term in the mathematical formulation of aerosol transport represents turbulent mixing.

6. For an air temperature=293 K, what is the altitude above the Earth's surface of an aerosol layer at 670 mb while the surface pressure is 1000 mb?

ANSWER

From hydrostatic approximation $dp = -\rho g dz$, and equation of state $p = \rho R_d T$, we can calculate

the altitude above ground $z = \int_{p_1}^{p_2} \frac{R_d T d \ln p}{g}$, with $R_d = 287 \text{ J.kg}^{-1}.\text{K}^{-1}$. Thus $z = \frac{R_d T}{g} \ln\left(\frac{p_2}{p_1}\right) =$

3400 m, with $p_1 = 670 \text{ mb}$, $p_2 = 1000 \text{ mb}$, $T = 293 \text{ K}$.

7. What is the percentage change of solar radiation reaching a ground surface characterized by an albedo $R_s = 0.2$ in the presence of a purely forward scattering ($\omega = 1$, $\beta = 1$) aerosol layer of optical thickness $\tau = 1$ when the solar zenith angle $\chi = 0$?

ANSWER

The total fraction of incident radiation transmitted to the ground is $t = e^{-\tau} + \omega(1 - \beta)(1 - e^{-\tau})$.

The reflected radiation tR_s which is partly scattered back to the ground is trR_s , and ultimately the fraction of incident light reaching the ground is

$$t + trR_s + tr^2R_s^2 + tr^3R_s^3 + \dots \approx \frac{t}{1 - rR_s} \text{ with } r = (1 - e^{-\tau})\omega\beta$$

For $\tau = 1$, $\omega = 1$, $\beta = 1$, the change of solar radiation reaching the ground is

$$\Delta R = \frac{e^{-\tau}}{1 - R_s(1 - e^{-\tau})} = \frac{0.368}{1 - 0.2 * (1 - 0.368)} = 0.42, \text{ which corresponds to a 58\% reduction of solar radiation.}$$

8. What are the variables you will need in order to calculate the altitude above ground given that the output are provided in sigma levels? Start by writing the formula relating z to pressure and then substitute with σ .

ANSWER

$\sigma = \frac{p - p_t}{p_s - p_t}$ and $dp = -\rho g dz$. So, $dz = (p_s - p_t) \frac{R_d T}{g} d\sigma$, and you would need the variables $p_s(x, y, t), T(x, y, \sigma, t)$ and the value of the constants g, R_d, p_t